

$C^{\frac{1}{2}}P^{\frac{1}{2}}T^{\frac{1}{2}}$ SYMMETRY OF THE CHAOTIC INFLATION DYNAMICS

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Abstract

In this work is shown that standard dynamical equations of the chaotic inflation for a massive real scalar field hold an especial local gauge symmetry. Given symmetry admits that inflation field mass can be considered as an especial "charge". Also, it is shown that given equations except expected CPT symmetry, hold surprisingly, $C^{\frac{1}{2}}P^{\frac{1}{2}}T^{\frac{1}{2}}$ symmetry too.

Key words: chaotic inflation - massive scalar field - CPT symmetry

In this work will be shown that standard dynamical equations of the chaotic inflation for a massive real scalar field hold an especial, local gauge symmetry. Given symmetry admits that inflation field mass can be considered as an especial "charge". Also, it is shown that given equations except CPT symmetry, hold $C^{\frac{1}{2}}P^{\frac{1}{2}}T^{\frac{1}{2}}$ symmetry too.

As it is well-known [1]-[7], chaotic inflation dynamics for a real scalar field with mass m can be generally presented by two usual differential equations

$$\frac{d^2\phi}{dt^2} + 3\frac{1}{a}\frac{da}{dt}\frac{d\phi}{dt} = -m^2\phi \quad (1)$$

$$\left(\frac{1}{a}\frac{da}{dt}\right)^2 + \frac{k}{a^2} = \frac{4\pi}{3m_P^2}\left(\left(\frac{d\phi}{dt}\right)^2 + m^2\phi^2\right). \quad (2)$$

Here ϕ represents the real scalar field depending of the time t , m_P - Planck's mass, a - scale factor of the universe, k - curvature constant (that equals 1 for closed, 0 for flat and -1 for open universe).

Define the following operator

$$D_t = \frac{d}{dt} - im \quad (3)$$

representing an especial covariant differentiation so that it is satisfied

$$D_t(e^{imt}\phi) = e^{imt}\frac{d\phi}{dt}. \quad (4)$$

By use of (3), (4) equation (1) can be presented in an equivalent form

$$D_t D_t(e^{imt}\phi) + 3\frac{1}{a}\frac{da}{dt}D_t(e^{imt}\phi) = -m^2(e^{imt}\phi) \quad (5)$$

or after complex conjugation, $*$, in other equivalent form

$$D_t^* D_t^*(e^{-imt}\phi) + 3\frac{1}{a}\frac{da}{dt}D_t^*(e^{-imt}\phi) = -m^2(e^{-imt}\phi) \quad (6)$$

Also, by use (3), (4) equation (2) can be presented in an equivalent form

$$\left(\frac{1}{a}\frac{da}{dt}\right)^2 + \frac{k}{a^2} = \frac{4\pi}{3m_P^2}(D_t(e^{imt}\phi)D_t^*(e^{-imt}\phi) + m^2(e^{imt}\phi)(e^{-imt}\phi)) \quad (7)$$

that does not change by complex conjugation.

It is not hard to see that equations (1), (2) and (5), (7) (or (6), (7)) are not only equivalent but they have equivalent forms. It simply means that dynamics of the chaotic inflation with massive scalar field is invariant, i.e. symmetric in respect to an especial local gauge transformation of the scalar field

$$\phi \rightarrow e^{imt}\phi \quad (8)$$

or complex conjugated transformation

$$\phi \rightarrow e^{-imt}\phi \quad (9)$$

It can be observed that transformation

$$m \rightarrow -m \quad (10)$$

does not change (1) and (2) as well as (7). On the other hand the same transformation changes (5) in (6) and vice versa. Generally speaking dynamics of the chaotic inflation with massive scalar field is invariant, i.e. symmetric in respect to transformation (10). But, according to (8), (9) it is obvious that transformation (10) can be compensated by complex conjugation, or roughly speaking, that (10) corresponds to complex conjugation. It opens an interesting possibility. Namely, according to characteristics of charge conjugation transformation, C , in quantum field theory [8], [9], mass of the inflationary quantum field can be effectively treated as a "charge". Simultaneously, symmetry of the dynamics of the chaotic inflation with massive scalar field in respect to (10) can be effectively considered as an especial type C symmetry. In this sense dynamics of the chaotic inflation with massive scalar field is C symmetric.

Further, time reversal transformation, T , [8], [9],

$$t \rightarrow -t \quad (11)$$

does not change (1) and (2) as well as (7). On the other hand the same transformation changes (5) in (6) and vice versa. It means that dynamics of the chaotic inflation with massive scalar field is T symmetric.

Finally, as it is well-known, demand for homogeneity of the space determine Robertson-Walker line element, which in the spherical coordinates, has form

$$ds^2 = -dt^2 + a^2\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right). \quad (12)$$

where

$$r^2 = x^2 + y^2 + z^2. \quad (13)$$

Obviously, parity transformation, P , [8], [9],

$$x \rightarrow -x \quad (14)$$

$$y \rightarrow -y \quad (15)$$

$$z \rightarrow -z \quad (16)$$

does not change (13) and (12) as well as dynamics of the chaotic inflation with massive scalar field (1), (2) or (5), (7) (or (6), (7)).

In this way it is shown that dynamics of the chaotic inflation with massive scalar field is C symmetric, P symmetric and T symmetric as well as CP , CT , PT and CPT symmetric.

Define additionally the following three transformations

$$m \rightarrow im \quad (17)$$

$$t \rightarrow it \quad (18)$$

$$k \rightarrow -k \quad (19)$$

where i represents the imaginary unit $(-1)^{\frac{1}{2}}$.

According to (10) and previous discussion (17) can be considered as $C^{\frac{1}{2}}$ transformation.

According to (11) transformation (18) can be considered as $T^{\frac{1}{2}}$ transformation.

Application of (19) and the following transformation

$$r \rightarrow ir \quad (20)$$

i.e.

$$x \rightarrow ix \quad (21)$$

$$y \rightarrow iy \quad (22)$$

$$z \rightarrow iz \quad (23)$$

representing, obviously, $P^{\frac{1}{2}}$, at $a^2(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2)$ changes given expression in an equivalent expression $-a^2(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2)$. In this way transformation (19) can be compensated by transformations (21)-(23), i.e. $P^{\frac{1}{2}}$, or, roughly speaking, transformation (19) can be considered equivalent to transformations $P^{\frac{1}{2}}$.

It is not hard to see that none of the equations (1)-(7) is invariant, i.e. symmetric in respect to $C^{\frac{1}{2}}$, $P^{\frac{1}{2}}$, $T^{\frac{1}{2}}$, $C^{\frac{1}{2}}P^{\frac{1}{2}}$, $C^{\frac{1}{2}}T^{\frac{1}{2}}$ and $P^{\frac{1}{2}}T^{\frac{1}{2}}$. Nevertheless, as it is not hard to see too, any of the

equations (1)-(7) is invariant, i.e. symmetric in respect to $C^{\frac{1}{2}}P^{\frac{1}{2}}T^{\frac{1}{2}}$. In this way it is shown that dynamics of the chaotic inflation with massive scalar field is $C^{\frac{1}{2}}P^{\frac{1}{2}}T^{\frac{1}{2}}$ symmetric.

In conclusion it can be shortly repeated and pointed out the following. In this work it is shown that standard dynamical equations of the chaotic inflation for a massive real scalar field hold an especial, local gauge symmetry. Given symmetry admits that inflation field mass be considered as an especial "charge". Also, it is shown that given equations except expected CPT symmetry, hold, surprisingly, $C^{\frac{1}{2}}P^{\frac{1}{2}}T^{\frac{1}{2}}$ symmetry too. All this represent an interesting result whose detailed discussion goes over basic intentions of this work.

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